NUMERICAL MODELLING OF INDUCED CONVECTION ABOVE SUBDUCTING SLABS

L. BODRI and B. BODRI

Department of Geophysics, Eötvös University, Budapest

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РЕЗЮМЕ

В работе проведено численное исследование потока вещества, вызванного движением опускающейся литосферной плиты. Математическое моделирование этого потока проводится в остром углу между горизонтальной литосферной плитой и опускающейся литосферой. Расчеты вынужденного потока двумерные, зависящие от времени, с вязкостью, зависящей от температуры. Расчеты очень чувствительны к реологическим свойствам материала в районе схождения двух литосферных плит. Моделирование вулканизма островных дуг, скорее всего, не может быть осуществлено без учета нелинейной реологии материала, а именно, прочности, пластичности и т. д. Вынужденный поток, исследованный в данной работе, является возможным объяснением вулканизма островных дуг, а также высокого теплового потока окраинных бассейнов.

Introduction

It is now well established in plate tectonics that the oceanic plates, continuously created at midoceanic ridges, descend into the mantle at ocean trenches as sketched in Figure 1. However, some of the processes occuring beneath such trenches and associated island arc – marginal basin systems have not yet been understood in all details. The observed large gravity anomalies (see in Fig. 2) near the trench systems are often attributed to the higher density of the relatively cold descending oceanic plates, however, it seems to be more probable that the regional positive anomalies extending from the island arc to, at least, the adjacent continental margin are produced by active upward emplacement of mantle material behind the subducting oceanic plates. The sometimes very detailed heat flow data of island arc areas show a very characteristic behaviour as presented in Figure 2. The heatflux from the oceanic plate as it entrers the subduction zone is sub-normal, there is a small decrease in the trench and the arc-trench gap but when the volcanic arc is reached the heat flow increases rapidly and relatively high values conserve for long distances behind the arc. At present there are two different approaches to the heat generation problem. T u r c o t t e and O x b u r g h (1968, 1970) and others (e.g. M i n e a r and T o k s ö z, 1970) have attributed the high surface heat flow to heat generation by viscous dissipation along a thin slip zone

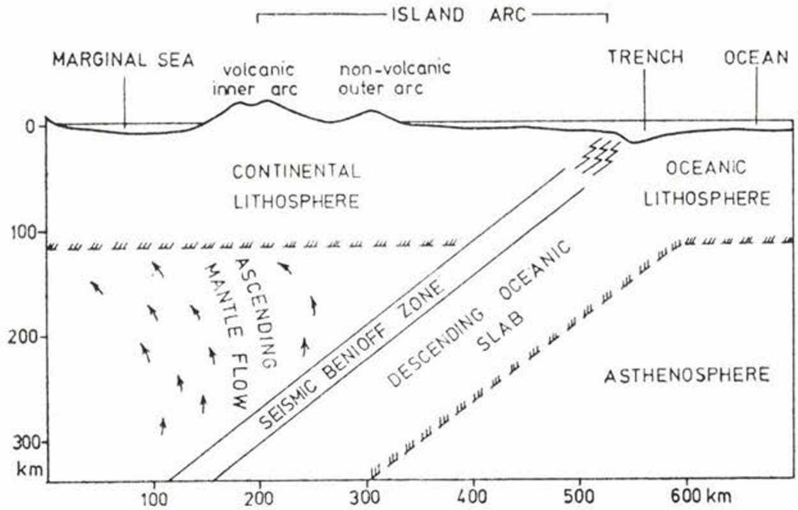


Fig. 1. Schematic cross-section of an island arc area.

Рис. 1. Схематический профиль района островной дуги

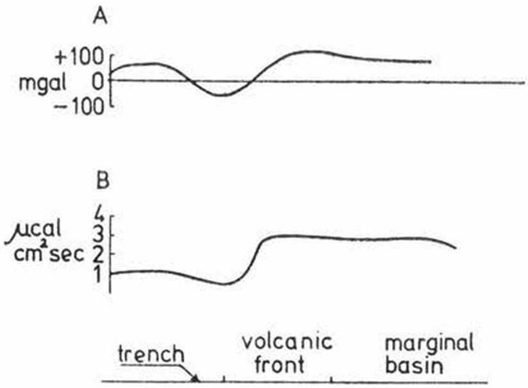


Fig. 2. Typical gravity anomaly distribution (A) and heat flow profile (B) in island arc-marginal basin systems.

Рис. 2. Типичный профиль гравитационных аномалий (A) и теплового потока (B) в системе островная дуга- окраинный бассейн

between the descending slab and the overriding mantle. McKenzie and Sclater (1968) pointed out that that stress or frictional heating in the Benioff zone alone would require unrealistic temperatures (7000°C) and a long time for heat transfer to the surface (300 m. y.) for producing the observed heat flow behind the arcs. According to Andrews and Sleep (1974) in the case of the mechanism of frictional heating the region above the slab must be fluid enough to convect termally, and at the same time yet so viscous that the descending slab can give rise to stress of about 2 kilobars. An other approach to the problem is the idea of A n d r e w s and S l e e p (1974) and others (e. g. I s a c k s, O l i v e r and S y k e s, 1968; M c K e n z i e, 1969), which suggests that high heat flow in marginal seas and island arc volcanism may be explained by a mechanically induced circulation of material within the wedge between the two lithospheric plates. In this paper we investigate this later process with numerical calculations.

Geophysical model and numerical results

The hydrodynamic modelling of material flow in the wedge region between the slab and the adjacent continental lithospheric plate is described below. Our model is in general similar to that of A n d r e w s and S l e e p (1974), however, there are also some significant differences. Figure 3, a schematic diagram of the area of the convergent plate margins, shows the fixed landward plate, the moving slab, a thin slip zone between the two lithospheric plates and two wedge-shaped regions separated by

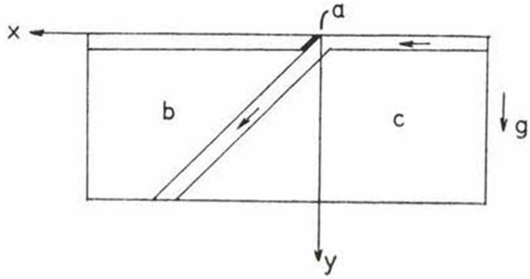


Fig. 3. Geometry of the model.

Рис. 3. Геометрия математической модели

the slab. It is obvious that only some particular regions of this complex system may be considered as viscous fluid. Observations of different types suggest that the independent lithospheric plates should be considered as rigid bodies affected by deformations only in thin zones near to their boundaries. This conclusion significantly determines the choosing of probable rheological models of the lithosphere and is against considering it as viscous fluid. According to hydrodynamical considerations, in case of typical fluids the hydrodynamic equations may be applied to processes the periods of which are much greater than molecular times. When a very viscous fluid is considered the situation is quite different; these equations are not more valid even in case of much greater periods of motion. According to L a n d a u and L i f s h i t z (1953) if the period of any force $\frac{1}{\omega}$ is large in comparison with the τ relaxation time, i. e. $\omega\tau \ll 1$, then the considered fluid manifests itself as a typical viscous one. The τ relaxation time for such fluids may be obtained from the following relation

$$\eta \sim \tau \mu , \tag{1}$$

where η is viscosity and μ is shear modulus. Taking $\mu = 0.5 \cdot 10^{12}$ CGS, we find that in our case (the time step in the numerical procedure is 10^6 y) the hydrodynamic equations may be valid up to viscosities $\eta = 10^{24} - 10^{25}$ poise. Therefore, as the calculation proceeded in time, the top of the fluid region might not lie above the actual isoline of viscosity of 10^{24} poise.

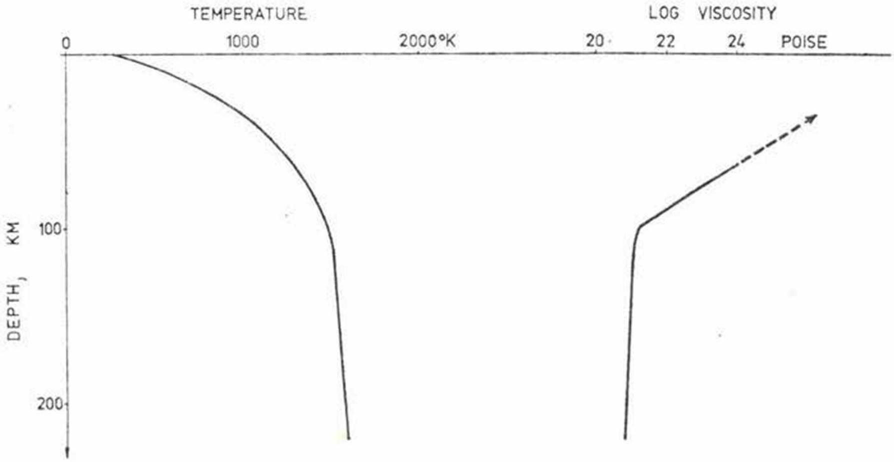


Fig. 4. Temperature and viscosity distribution in the initial, laterally uniform state.

Рис. 4. Распределение температуры и вязкости в первоначальной горизонтально однородной стадии

Assuming an initial, laterally uniform temperature distribution as shown in Figure 4, the corresponding viscosity profile can be calculated by the simple relation (Zharkov et al., 1971)

$$\eta = \eta_0 \exp \frac{E}{RT} , \tag{2}$$

where T is temperature in deg K, $\eta_0 = 1.104 \cdot 10^8$ poise and $\frac{E}{R} = 4.758 \cdot 10^4$ deg. So, in the initial state region b with an upper boundary depth of 60 km is fluid enough to convect thermally, while the overriding layer should be considered as a solid. Since the thermal conductivity of the down-going slab is assumed to be negligibly small, the zero heat flux boundary condition should be used on its surface. It means that the moving slab will retain its originally low temperature even after it has sunk to considerable depths. Since the maximal temperatures within the slab are not more than

700 – 800°C (McKenzie, 1969), it can be considered as a solid during the whole period of calculation. So, the problem is the modelling of induced convection within a fluid wedge region bounded by two solid plates. The solution of the task, however, is rather problematic, since it was recognized in hydrodynamics long ago that sharp changes in the form of a surface boundaring any fluid region (in our case it is the corner of the wedge) give rise to large, even to infinite local stresses. The infinite incresae of stresses at such places is due to the more complicated (non-linear) rheological properties of material, i.e. the linear Hooke's law used for the determination of stesses is not more valid here. The discussion of causes of concentration of stresses is out of the scope of the present paper, we note only that in similar to this models it should be always taken into account that the lithospheric plates are characterized by τ_s finite yield stresses and while the stresses arising during any process do not exceed τ_s the plates remain elastic, or (in the first approximation) rigid bodies. An elementary calculation shows, however, that in our case the above mentioned non-linear rheological behaviour of material will take place in a zone of quite small extent, away from the corner the stresses will decrease rapidly. In the present study we assumed that in the region where the stresses exceed the critical value the lithosphere will transform to plastic state with properties similar to those of a viscous fluid of some η_e effective viscosity. This is equivalent to changing the lower boundary of the solid landward plate during the process of calculation. The critical stresses will, of course, exercise an influence also on the slab but because of the motion always new and new parts of the sinking slab will be affected by a certain level of stresses. Therefore its boundary was considered as unchanged during the whole period of calculation. A thin slip zone (denoted by a in Fig. 3) filled with oceanic crust is included between the two plates. The rocks of the oceanic crust are characterized by significantly less yield stresses than those of the lithosphere, therefore entered the zone of friction of the two plates it must entirely transform to plastic state with rheology similar to that of a viscous fluid included between the surface of two solids moving relatively to each other. The thermal and mechanical state of this zone has been investigated in details by a number of authors (e. g. Turcotte and Oxburgh, 1968; Turcotte and Schubert, 1973; Jischke, 1975). The horizontal extent of the slip zone, however, is much less than that of our calculational region, therefore it appeared in our model as a thermal boundary condition on the solid landward plate. The induced tectonic flow in wedge b has been calculated on the basis of equations of viscous motion and of thermal conduction. The two-dimensional equation of motion was taken in the following form:

$$\frac{\partial^2}{\partial x\,\partial y}\left[4\eta\frac{\partial^2 S}{\partial x\,\partial y}\right]+\left(\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial x^2}\right)\left[\eta\left(\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial x^2}\right)S\right]+$$

$$+\varrho_0 g_0 \alpha\frac{\partial T}{\partial x}=0\,, \qquad (3)$$

where S is stream function, T is temperature, η is viscosity, α is thermal expansivity, ϱ_0 is density and g_0 is gravity acceleration. The equation above is in Boussinesq approximation which means that only the variation of density with temperature need be considered, and only in the body-force term. The equation of heat conduction is

$$\frac{\partial T}{\partial t} = \left(\frac{\partial S}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial S}{\partial x}\frac{\partial T}{\partial y}\right) + \frac{1}{\varrho_0 c_p}\left[\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + \right.$$
$$\left. + 4\eta\left(\frac{\partial^2 S}{\partial x\,\partial y}\right)^2 + \eta\left(\frac{\partial^2 S}{\partial y^2} - \frac{\partial^2 S}{\partial x^2}\right)^2 + H\right], \qquad (4)$$

where K is thermal conductivity and H is the rate of heat generation per unit volume. The adiabatic temperature changes have not been taken into account in equation (4).

The question of boundary conditions arises well known difficulties. Equation (3) has been solved in the wedge bounded by the slab and the landward plate. As already mentioned above, the lower boundary of the island arc plate was determined by isoline of $\eta = 10^{24}$ poise. The boundary conditions on the top of the wedge are

$$S = 0$$
$$\text{and } \vec{v} = 0,$$
$$\text{where } \vec{v} = \left\{\frac{\partial S}{\partial y}\,;\ -\frac{\partial S}{\partial x}\,;\ 0\right\}. \qquad (5)$$

On the slab boundary we have

$$S = 0$$
$$\text{and } \vec{v} = \vec{v}_0, \qquad (6)$$

where $\vec{v}_0$ is the slab velocity, parallel to the surface of the slab which is dipped in our case at 45°. There are no lower and left-hand side boundaries to the wedge in our model. In order to be able to obtain solution a re-zoning procedure was needed as it has been applied by A n d r e w s and S l e e p (1974). The calculation was started with a grid extending horizontally 800 km from the corner and extending to a depth of 400 km with zones of 20×20 km. The values of stream function on the critical boundaries of this grid were available from the analytic solution of B a t c h e l o r (1967). After having obtained the solution on the coarse grid, a finer grid extending to 600×300 km with zones 10 by 10 km was used. Boundary values for the stream function were taken from solution related to the coarse grid. The equation of the heat conduction has been solved in the whole region behind the slab. The thermal boundary conditions are

$$T(y=0) = 273°\text{K} \qquad (7)$$

throughout the free surface of the landward plate and

$$\frac{\partial T}{\partial n} = 0 \tag{8}$$

at all points of the astenosphere-slab boundary. Here $\frac{\partial}{\partial n}$ denotes differentiation in the direction of the outward normal to the surface of the slab. The temperature at each point on the surface of the slab was assumed to be equal to that of the fluid astenosphere, obtained from equation (4) with the boundary consition given above. The zero heat flux boundary condition however, is not more valid along the line of contact of the two lithospheric plates, since a thin slip zone of considerable conductivity is located between them. It was already mentioned above that because of the small horizontal extent of this zone, its presence has only been taken into account as a temperature boundary condition on the landward plate. The temperature on the right boundary of the solid landward plate was assumed to be constant and equal to 800°C.

Besides the stream function the stress components also have been computed during the iterative procedure. A yield condition was introduced according to which the stress components at the boundaries should not exceed some $s_{\max}$ given value equal to 1 kb in our case. This condition can be formulated as

$$|s_{xx}| \leq s_{\max}, \quad |s_{yy}| \leq s_{\max}. \tag{9}$$

If inequalities (9) are not satisfied, then the values of viscosity found from the constitutive relation are replaced by some effective viscosity and the boundary is changed correspondingly. The temperatures are computed only if equality holds in expressions (9) after convergence is achieved in the iterative procedure for equation (3). If the yield condition is not satisfied at any point of the grid, then some effective temperature corresponding to effective viscosity η_e is introduced there.

The calculated streamlines and isotherms are shown in Figures 5, 6 and 7 at 1.0, 4.0 and 7.0 My from start of subduction respectively The following numerical values of parameters are used throughout these calculations:

$$\begin{aligned}
\varrho_0 &= 3.4 \text{ g cm}^{-3} \\
g_0 &= 990 \text{ cm sec}^{-2} \\
\alpha &= 4 \cdot 10^{-5} \text{ deg}^{-1} \\
c_p &= 0.311 \text{ cal g}^{-1} \text{ deg}^{-1} \\
\eta_e &= 1 \cdot 10^{24} \text{ poise.}
\end{aligned}$$

The assumed dependence of thermal conductivity on temperature is

$$\lambda = a + b\,T^3,$$

where $a = 6 \cdot 10^{-3}$ cal cm^{-1} sec^{-1} deg^{-1} and

$b = 5 \cdot 10^{-12}$ cal cm^{-1} sec^{-1} deg^{-1}.

The viscosity was calculated by formula (2), the slab dipped at 45° and was given a constant velocity of 10 cm yr^{-1}. The temperature and the corresponding viscosity distribution in the initial state are laterally uniform. At 1.0 My from start of calculation the solution for the stream function does not deviate significantly from Batchelor's wedge solution. Near to the slab boundary, however, a hot zone appears with low viscosity. Stresses arising at the corner, begin to exceed the critical value already at the first steps of calculation. The horizontal stress near the corner is compressive, the region of large stresses extends approximately to 40 km both along

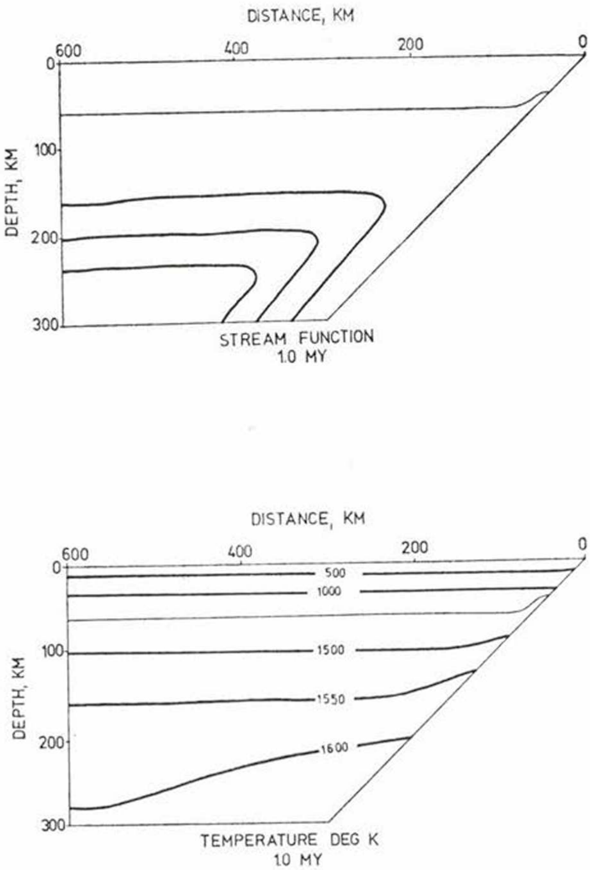
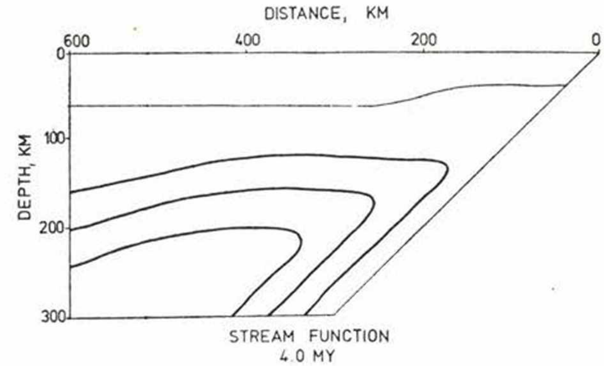


Fig. 5. Streamlines and isotherms at 1.0 My from the beginning of subduction.

Рис. 5. Линии тока и изотермы через 1.0 млн. лет после начала субдукции



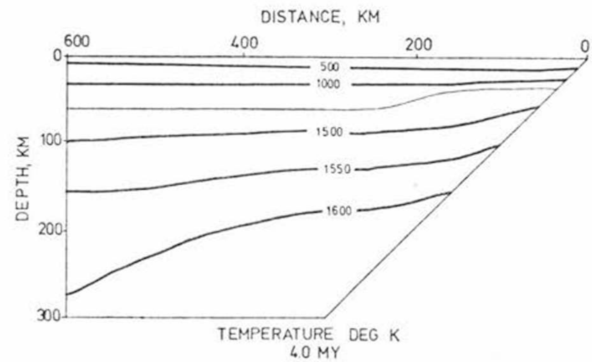
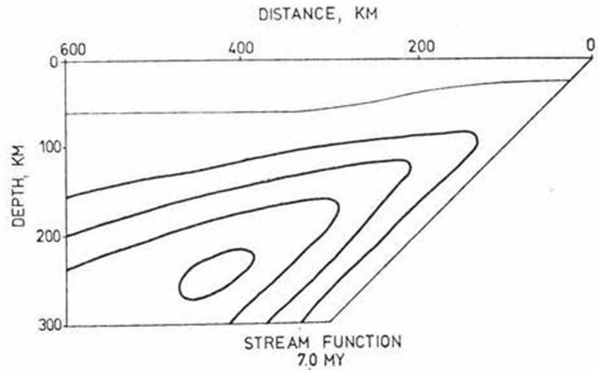


Fig. 6. Streamlines and isotherms at 4.0 My.

Рис. 6. Линии тока и изотермы в 4.0 млн. лет

the horizontal plate and the slab boundary. The lithosphere somewhat thinned at 1.0 My only in the zone of immediate contact of the two plates. As time progresses this zone spreads, so that at 7.0 My the region of thinned lithosphere extends up to 100 km. This result is quite realistic, since all of the stress components may be expressed as biharmonic functions and according to a fundamental property of these functions, the maximal values of stress components are reached at the boundaries of the considered region. As it expected, the streamlines dublicate the shape of the boundaring surfaces. As time proceeds, the temperature within the convecting region increases continuously, isotherms become nonhorizontal while within the plate they remain horizontal and there are not significant changes of temperature. The calculated heat flow profile above the region at 7.0 My is shown in Fig. 8. There is a high heat flow in the zone of the thinned lithosphere and it decreases away from this zone in both directions.



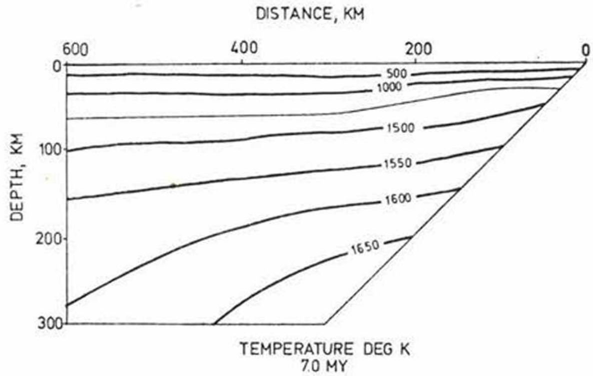


Fig. 7. Streamlines and isotherms at 7,0 My.

Рис. 7. Линии тока и изотермы в 7.0 млн. лет.

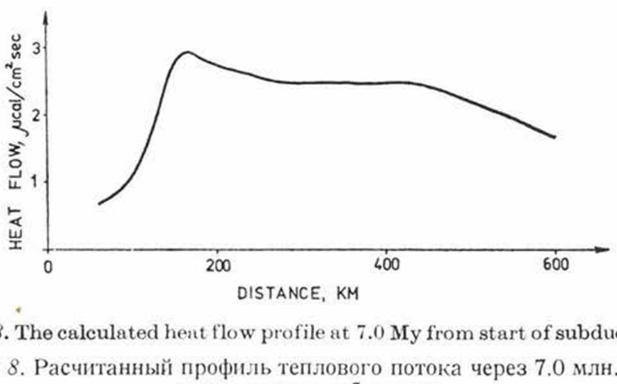


Fig. 8. The calculated heat flow profile at 7.0 My from start of subduction.

Рис. 8. Расчитанный профиль теплового потока через 7.0 млн. лет после начала субдукции

Remarkable similarity exists between this calculated profile and the measured one (see Fig. 2).
The following model of island arc volcanism is supported by these calculations. The volcanic arc might occur closely above the corner of the slab with the horizontal lithosphere. Due to changes in rheology of material part of the lithosphere can become thinned, a certain portion of material of the landward plate is given the possibility to take part in the general convective motion that takes place in the wedge, so the portion of the plate adjacent to the slab is slowly subducted with the slab. As time progresses, the zone of thinned lithosphere expands so, that new volcanic arc may also occure at relatively larger distances away from the corner. The zone where the lithosphere continuously gets thicker is characterized by decreasing of the heat flow from high to normal values. Upwelling flow in this zone may in principle, induce spreading, somewhat similar to that at a mid-ocean ridge but it is non symmetrical here.

Conclusions

The wedge between the horizontal lithosphere plate and the subducting slab is fluid enough that induced convection can take place there for time of order of magnitude of My. The lithosphere plates should be considered as solids and at the zones of critical stresses non-linear rheology is to be applied.
Critical stresses arise near sharp changes of shapes boundaring the convecting fluid region. As time progresses, these zones will be extending and stresses certainly exceed in this region the yield stress of lithosphere.
The whole island arc area extends horizontally to some hundered kms, i. e. it almost entirely located in the zone of large stresses so it seems probable that models of island arcs can not be constructed without taking into account non-linear processes, like plasticity, yielding etc. The modelling of these later processes, however, is an extremely difficult task, the way suggested here for including them into the model is probably a crude approximation, however, it leads to realistic results in the greater, part of the investigated area.
However, even by this idealized modelling of the effects of lithisphere-astenosphere interactions some of processes occuring beneath island arcs may be explained satisfactorily. It seems to be probable that elasto-plastic behaviour of lithosphere may be responsible for processes arising asymmetrically on both sides of the subduction zone, since stresses and consequently the effects of non-linear rheology arising on the corner of an obtuse-angled wedge probably significantly less then those on the other side of the slab.

REFERENCES

Andrews, D. J. and Sleep, N. H., 1974. Numerical modelling of tectonic flow behind island arcs. Geophys. J. R. Astron. Soc., 38: 237 – 251.

Batchelor, G. K., 1967. An introduction to fluid dynamics. Cambrige University Press, Cambrige, 224 pp.

Isacks, B., Oliver, J. and Sykes, L., 1968. Seismology and the new global tecnics. J. Geophys. Res., 73: 5855 – 5899.

Jischke, M. C., 1975. On the dynamics of descending lithospheric plates and slip zones J. Geophys. Res., 80: 5876 – 5886.

Landau, L. D. and Lifshitz, E. M., 1953. Mechanika sploshnih sred. GITTL., Moscow, 788 pp. (in Russian).

Minear, J. W. and Toksöz, M. N., 1970. Thermal regime of a downgoing slab and new global tectonics. J. Gephys. Res., 75: 1397 – 1419.

McKenzie, D., 1969. Speculations on the consequences and causes of plate motions. Geophys. J. R. Astron. Soc., 18: 1 – 32.

McKenzie, D. P. and Sclater, J. G., 1968. Heat flow inside the island arcs of the northwestern Pacific. J. Geophys. Res., 73: 3173 – 3179.

Oxburgh, E. R. and Turcotte, D. L., 1968. Problem of high heat flow and volcanism associated with zones of descending mantle convective flow. Nature, 218: 1041 – 1043.

Oxburgh, E. R. and Turcotte, D. L., 1970. Thermal structure of island arcs Geol. Soc. Am. Bull., 81: 1665 – 1688.

Turcotte, D. L. and Schubert, G., 1973. Frictional heating of the descending lithosphere. J. Geophys. Res., 78: 5876 – 5886.

Zharkov, V. N., Trubicin, V. P. and Samsonienko, L. V., 1971: Fizika planet i Zemli. Nauka, Moscow, 383 pp. (in Russian).